

APPROXIMATIONSRelative error, percentage error and Tolerance

The relative error of a dimension is the ratio of the absolute error to the true value.

$$\text{Relative error} = \frac{\text{absolute error}}{\text{true value}}$$

The percentage error states the relative error as a percentage.

$$\text{percentage error} = \text{relative error} \times 100\%$$

Tolerance is the difference between the greatest and the least acceptable measurements.

Example

A mass of 24 kg is recorded as 24.3 kg. Find

- the relative error
- the percentage error

solution

$$\begin{aligned} \text{(a) absolute error} &= |\text{recorded value} - \text{true value}| \\ &= |24.3 - 24| \\ &= 0.3 \text{ kg} \end{aligned}$$

$$\begin{aligned} \text{relative error} &= \frac{\text{absolute error}}{\text{true value}} \\ &= \frac{0.3 \text{ kg}}{24 \text{ kg}} \\ &= 0.0125 \end{aligned}$$

$$\begin{aligned}
 \text{(b) percentage error} &= \text{relative error} \times 100\% \\
 &= 0.0125 \times 100\% \\
 &= 1.25\%
 \end{aligned}$$

Example

Find the percentage error of the mass of a bag of sugar that weighs 10.00 kg, correct to 2 decimal places.

Solution

$$\text{least unit of measurement} = 0.01 \text{ kg}$$

$$\text{absolute error} = \frac{n}{2} = \frac{0.01}{2} = 0.005 \text{ kg}$$

$$\text{relative error} = \frac{\text{absolute error}}{\text{true value}}$$

$$= \frac{0.005 \text{ kg}}{10.00 \text{ kg}}$$

$$= 0.0005$$

$$\text{percentage error} = \text{relative error} \times 100\%$$

$$= 0.0005 \times 100\%$$

$$= 0.05\%$$

Example

Find the tolerance in each of the following:

(a) The upper limit is 7m and the lower limit is 3m.

(b) the measurements lie between (7.5 ± 1.3) h

Solution:

(a) Tolerance = $(7 - 3)m = 4m$

(b) Tolerance = $(7.5 + 1.3)h - (7.5 - 1.3)h$
 $= 8.8h - 6.2h$
 $= 2.6h$

Exercise

- The true value of the length of a rectangle is 4 cm. If this is recorded as 4.4 cm, find:
 - the absolute error
 - the relative error
 - the percentage error
- The length of a piece of wire is 15.2 cm, correct to 1 decimal place. What is the relative error of the length of the piece of wire?
- A bag of potatoes has mass (15.4 ± 0.05) kg
 - Find the tolerance of this mass
 - write down the relative error of the mass as a fraction in its simplest form.
- The length of a piece of wire is measured as 4.5 cm. Calculate
 - the tolerance
 - the relative error