11C / 11E

SEQUENCES AND SERIES

THE SUM OF AN ARITHMETIC PROGRESSION

An arithmetic series is the sum of an arithmetic progression. So, an arithmetic series is an AP with plus signs written between the terms.

For example, if we have an arithmetic progression 1, 4, 7, 10, 13, ..., then the arithmetic series is 1+4+7+10+13+...

The symbol S_n is used to represent an arithmetic series.

The formula for the sum of the first n terms in an AP, where a is the first term and d is the common difference, is:

$$S_n = \frac{n}{2}[2a+(n-1)d]$$

If the last term of an AP is known, then we can use the formula

$$S_n = \frac{n}{2}[a+L]$$

where a is the first term, n is the number of the terms in the AP and L is the last term.

EXAMPLE

1. Calculate the sum of the first 50 terms in the AP: 1, 3, 5, 7, . . .
2. Calculate the sum of the first 65 terms in the AP: -1, -3, -5, . . .
3. Calculate the sum of the AP 2, 4, 6, . . ., 102
4. Calculate the sum of the AP -50, -47, -44, . . ., -8

Solutions

1. 1, 3, 5, 7, . . .

$a = 1$, $n = 50$, $d = T_2 - T_1 = 3 - 1 = 2$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{50} = \frac{50}{2}[2(1) + (50-1)(2)]$$

$$= 25[2 + (49)(2)]$$

$$= 25[2 + 98]$$

$$= 25[100]$$

$$= \underline{2500}$$

2. -1, -3, -5, . . .

$a = -1$, $n = 65$, $d = T_2 - T_1 = -3 - (-1) = -3 + 1 = -2$

$$S_n = \frac{n}{2}[2a + (n-1)d]$$

$$S_{65} = \frac{65}{2}[2(-1) + (65-1)(-2)] = \frac{65}{2}[-2 + (64)(-2)]$$

$$= \frac{65}{2}[-2 - 128] = \frac{65}{2}[-130] = \underline{-4225}$$

3. $2, 4, 6, \ldots, 102$

First we find the number of terms in the AP

$T_n = a + (n-1)d$ $\qquad a = 2,\ d = T_2 - T_1 = 4 - 2 = 2$

$102 = 2 + (n-1)2$ $\qquad n = ? \qquad T_n = 102$

$102 = 2 + 2n - 2$

$102 = 2 - 2 + 2n$

$102 = 2n$

$\frac{102}{2} = \frac{2n}{2}$

$51 = n$

$n = 51$

Therefore there are 51 terms in the AP

Since the last term is known, we can use the formula $S_n = \frac{n}{2}[a + L]$

$a = 2,\ n = 51,\ L = 102$

$\therefore \quad S_n = \frac{n}{2}[a + L$

$S_{51} = \frac{51}{2}[2 + 102] = \frac{51}{2}[104] = 51 \times 52 = \underline{\underline{2652}}$

4. $-50, -47, -44, \ldots, -8$

First we find the number of terms in the AP

$T_n = a + (n-1)d$ $\qquad a = -50,\ d = T_2 - T_1 = -47 - (-50)$

$-8 = -50 + (n-1)3$ $\qquad T_n = -8 \qquad = -47 + 50$

$\qquad = 3$

$-8 = -50 + 3n - 3$ $\qquad n = ?$

$-8 = -50 - 3 + 3n$

$-8 = -53 + 3n$

$53 - 8 = 3n$

$45 = 3n$

$\frac{45}{3} = \frac{3n}{3}$

$15 = n$

$n = 15$

Therefore there are 15 terms in the AP

Since the last term is known, we can use the formula $S_n = \frac{n}{2}[a + L]$

$a = -50$, $n = 15$, $L = -8$

$$S_n = \frac{n}{2}[a + L]$$

$$S_{15} = \frac{15}{2}[-50 + (-8)] = \frac{15}{2}[-58] = \underline{-435}$$

EXERCISE

1. Calculate the sum of the first 48 terms in the AP : 4, 8, 12, . . .

2. Calculate the sum of the first 15 terms in the AP = 11, 5, −1, −7, . . .

3. Calculate the sum of the AP

   3, 5, 7, . . ., 43

4. Calculate the sum of the AP

   −12, −10, −8, . . ., 32

5. Calculate the sum of the AP

   24, 16, 8, . . ., −56