

Subsets

A set B is a subset of a set A if every member of B is a member of A .

If $A=B$, then B is a subset of or equal to A and vice versa. We write this as $B \subseteq A$ or $A \subseteq B$.

A set is a subset of itself.

The empty set is a subset of every set.

Example

Given that $A = \{1, 2\}$, list all the subsets of A .

Solution:

$\{\}, \{1\}, \{2\}, \{1, 2\}$

Example

What are the subsets of set $B = \{a, b, c\}$?

Solution:

$\{\}, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}$ and $\{a, b, c\}$

Number of subsets

If there are n elements in a given set A , then there are 2^n subsets of set A .

Example

If $B = \{p, q, r, t\}$, find the number of subsets of B .

Solution:

$$\begin{aligned} \text{Number of subsets} &= 2^n, \text{ where } n=4 \\ &= 2^4 \\ &= 16 \end{aligned}$$

Therefore the number of subsets of B is 16.

Example

Set D has 512 subsets. How many elements does D have?

Solution:

$$\text{number of subsets} = 2^n$$

$$512 = 2^n$$

$$2^9 = 2^n$$

$$\therefore 9 = n$$

$$n = 9$$

Therefore set D has 9 elements.

proper subsets

A proper subset of a set A is a subset of A which contains at least one element but not all the elements of A. If B is a proper subset of A, we write $B \subset A$.

A set is not a proper subset of itself.

The empty set is not a proper subset of any set because it has no element.

Example

A set T has 30 proper subsets. How many elements does T have in it?

Solution:

Number of subsets of a set is given by the sum of its proper subsets plus the empty set and the set itself. (since every set is a subset of itself)

i.e. number of proper subsets + 2.

Therefore T has 32 subsets.

$$\therefore \text{number of subsets} = 2^n$$

$$32 = 2^n$$

$$2^5 = 2^n$$

$$\therefore 5 = n$$

$$n = 5$$

Therefore T has 5 elements

Exercise

1. Given that $M = \{\text{the first 3 natural numbers}\}$
 - (a) write all the subsets of M
 - (b) which subsets of M are not proper subsets?
2. Set P has 128 subsets. How many elements does P have?
3. A set Y has 14 proper subsets. How many elements does Y have in it?