12 A LESSON 1

VECTORS

A vector is a quantity that has both magnitude (size) and direction.

Representing and denoting vectors

If a line PQ represents a vector, the vector is written as $\overrightarrow{PQ}$.

vector $\overrightarrow{PQ}$

vectors are also labelled with a single letter, for example $\underline{a}$.

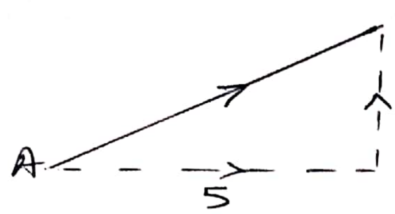
If AB is a line segment, then $\overrightarrow{AB}$ and $\overrightarrow{BA}$ have the same size and are parallel to each other, but have opposite directions,

So $\overrightarrow{BA} = -\overrightarrow{AB}$

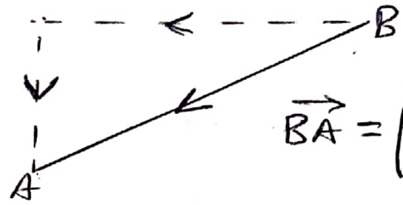
A vector can also be represented as a column matrix in the form $\begin{pmatrix} x \\ y \end{pmatrix}$. It is called a column vector.

Two vectors are equal if they have the same magnitude and direction, no matter what their initial point is.

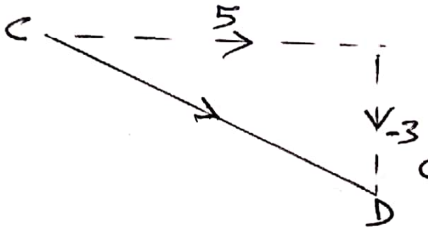
Direction of vectors



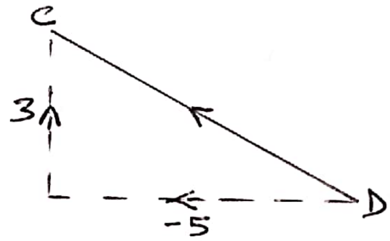
$\overrightarrow{AB} = \begin{pmatrix} 5 \\ 3 \end{pmatrix}$



$\overrightarrow{BA} = \begin{pmatrix} -5 \\ -3 \end{pmatrix}$



$\overrightarrow{CD} = \begin{pmatrix} 5 \\ -3 \end{pmatrix}$



$\overrightarrow{DC} = \begin{pmatrix} -5 \\ 3 \end{pmatrix}$

Magnitude of a vector

The magnitude of a vector $\overrightarrow{AB}$ is the length of the directed line segment AB. It is written as $|\overrightarrow{AB}|$ and is called the modulus of $\overrightarrow{AB}$.

If $\overrightarrow{AB} = \begin{pmatrix} x \\ y \end{pmatrix}$, then $|\overrightarrow{AB}| = \sqrt{x^2 + y^2}$

Example

If $\overrightarrow{AB} = \begin{pmatrix} -2 \\ -5 \end{pmatrix}$, find the magnitude of $\overrightarrow{AB}$.

Solution

$$|\overrightarrow{AB}| = \sqrt{(-2)^2 + (-5)^2} = \sqrt{4 + 25} = \sqrt{29} \text{ units}$$

Addition and Subtraction

If $\underline{p} = \begin{pmatrix} -9 \\ 0 \end{pmatrix}$ and $\underline{q} = \begin{pmatrix} 6 \\ -2 \end{pmatrix}$

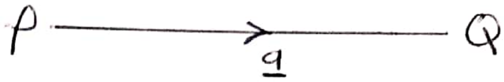
$$\underline{p} + \underline{q} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} + \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -9+6 \\ 0+(-2) \end{pmatrix} = \begin{pmatrix} -3 \\ -2 \end{pmatrix}$$

$$\underline{p} - \underline{q} = \begin{pmatrix} -9 \\ 0 \end{pmatrix} - \begin{pmatrix} 6 \\ -2 \end{pmatrix} = \begin{pmatrix} -9-6 \\ 0-(-2) \end{pmatrix} = \begin{pmatrix} -15 \\ 2 \end{pmatrix}$$

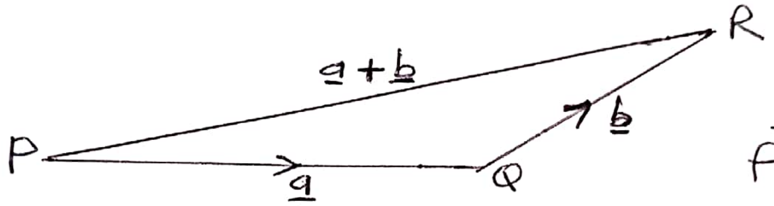
Triangle Rule of addition of vectors

Given

and



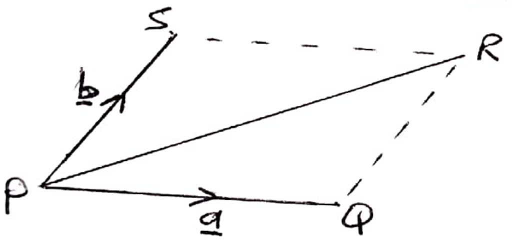
adding $\overrightarrow{PQ}$ and $\overrightarrow{QR}$ we have



$\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

The vector $\overrightarrow{PR}$ is called the resultant vector

Parallelogram Rule



If we apply the triangle rule we get

$\overrightarrow{PR} = \overrightarrow{PS} + \overrightarrow{SR}$ or $\overrightarrow{PR} = \overrightarrow{PQ} + \overrightarrow{QR}$

$= \underline{b} + \underline{a}$ $\qquad = \underline{a} + \underline{b}$

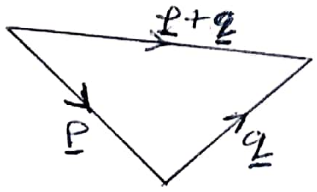
Example

Given the vectors $\underline{p}$ and $\underline{q}$ below

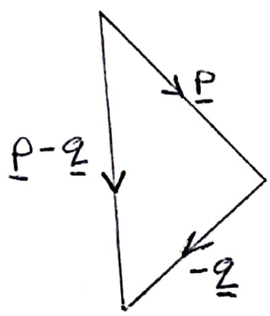
draw the vectors (i) $\underline{p} + \underline{q}$ (ii) $\underline{p} - \underline{q}$

Solution:

(i)



(ii)



$\underline{p} - \underline{q} = \underline{p} + (-\underline{q})$

Exercise

1. Write down a single vector equivalent of each of the following.

(a) $\overrightarrow{AB} + \overrightarrow{BC}$

(b) $\overrightarrow{NM} + \overrightarrow{MK} + \overrightarrow{KD}$

2. Complete the following so that the statements are always true:

(a) $\overrightarrow{IT} + \overrightarrow{TD} =$ ____

(b) $\overrightarrow{DJ} +$ ____ $= \overrightarrow{DN}$

(c) $\overrightarrow{CS} +$ ____ $+ \overrightarrow{TF} = \overrightarrow{CF}$

3. Given the following vectors

$\overrightarrow{PQ} = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$, $\overrightarrow{QR} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$, $\overrightarrow{PT} = \begin{pmatrix} 5 \\ 1 \end{pmatrix}$

and $\overrightarrow{ST} = \begin{pmatrix} -1 \\ -2 \end{pmatrix}$, use the triangle rule to find

(a) $\overrightarrow{PR}$ (b) $\overrightarrow{PS}$

4. Find the magnitude of each of the following vectors

(a) $\overrightarrow{AB} = \begin{pmatrix} 8 \\ 6 \end{pmatrix}$ (b) $\overrightarrow{BC} = \begin{pmatrix} 5 \\ 12 \end{pmatrix}$

(c) $\overrightarrow{PQ} = \begin{pmatrix} 3 \\ -1 \end{pmatrix}$