

$$\frac{a}{\sin 82^\circ} = \frac{6.7}{\sin 59^\circ}$$

Cross multiplying and making a the subject gives

$$a = \frac{6.7 \times \sin 82^\circ}{\sin 59^\circ} \text{ cm}$$

$$a = 7.740374668 \text{ cm}$$

Since c is given correct to 1 decimal place,

$$\therefore \underline{a = 7.7 \text{ cm}}$$

To solve for b, we can use either  $\frac{a}{\sin A}$  or  $\frac{c}{\sin C}$ .

$$\frac{a}{\sin A} = \frac{b}{\sin B}$$

$$\frac{7.7}{\sin 82^\circ} = \frac{b}{\sin 39^\circ}$$

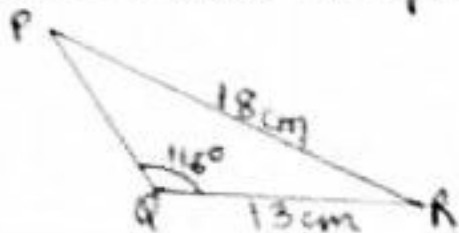
$$b = \frac{7.7 \sin 39^\circ}{\sin 82^\circ} \text{ cm}$$

$$\underline{b = 4.893389138 \text{ cm}}$$

$$\therefore \underline{b = 4.9 \text{ cm}}$$

### Example 2

Solve  $\triangle PQR$  completely.



### Solution

$$\frac{r}{\sin P} = \frac{p}{\sin R} = \frac{r}{\sin R}$$

$$\frac{13}{\sin P} = \frac{18}{\sin 116^\circ}$$

$$\sin P = \frac{13 \sin 116^\circ}{18}$$

$$\sin P = 0.649129033$$

$$P = \sin^{-1} 0.649129033$$

$$\therefore \underline{P = 40.5^\circ}$$

$$R = 180^\circ - (116^\circ + 40.5^\circ)$$

$$R = \underline{23.5^\circ}$$

$$\frac{p}{\sin P} = \frac{r}{\sin R}$$

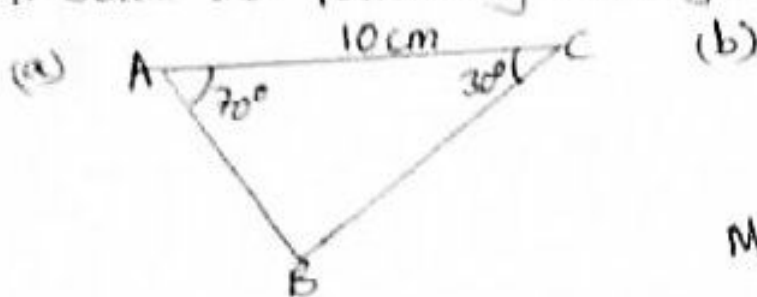
$$\frac{18}{\sin 116^\circ} = \frac{r}{\sin 23.5^\circ}$$

$$r = \frac{18 \sin 23.5^\circ}{\sin 116^\circ}$$

$$r = 7.985681781$$
$$\therefore r = \underline{8.0\text{cm}} \text{ or } r = \underline{8\text{cm}}$$

### EXERCISE

1. Solve the following triangles completely.



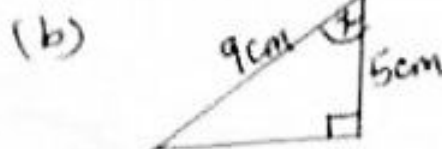
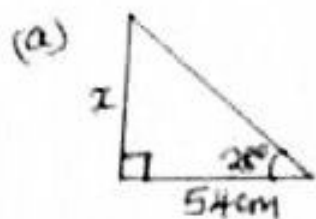
2. Solve

(a)  $\triangle PQR$ , if  $\angle Q = 124^\circ$ ,  $r = 16\text{cm}$  and  $q = 29\text{cm}$ .

(b)  $\triangle XYZ$ , if  $Y = 29.8^\circ$ ,  $Z = 51.4^\circ$  and  $x = 19.6\text{cm}$ .

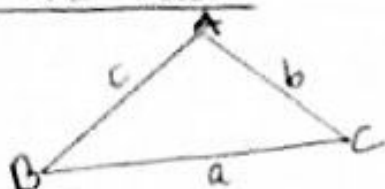
### PREP WORK

Calculate the value of  $x$  in each of the following triangles:



Non right-angled triangles

To calculate angles and sides of non right-angled triangles the Sine Rule and Cosine Rule are used.

The Sine Rule

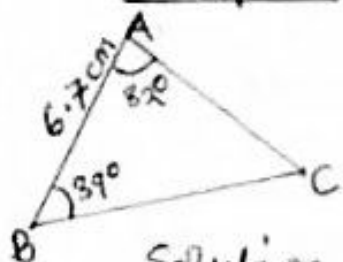
In the triangle ABC, the angles are denoted by Capital letters A, B and C and the sides opposite these angles by a, b and c respectively.

\* For any triangle ABC, the Sine Rule states that:

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C} \quad \text{or} \quad \frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

\* The Sine Rule is used to solve non right-angled triangles in which:

- two angles and one side are given
- two sides and the angle opposite to one of them are given.

Example 1

Solve triangle ABC completely.

Solution

\* To solve the triangle completely, means to calculate all the unknown angles and sides.

∵ all the angles in  $\triangle ABC$  added should give  $180^\circ$   
 $\therefore \angle C = 59^\circ$

$$\frac{a}{\sin A} = \frac{b}{\sin B} = \frac{c}{\sin C}, \quad a = ?, \quad b = ?, \quad c = 6.7$$